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Influence of variable viscosity of mineral oil on laminar heat transfer in a 2:1 rectangular duct†

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Abstract—An experimental study of laminar flow heat transfer in a 2:1 rectangular duct to mineral oil is carried out. The H1 thermal boundary condition corresponding to axially constant heat flux and peripherally constant temperature is adopted for three different heating configurations: (1) top wall heated, other walls adiabatic; (2) bottom wall heated, other walls adiabatic; (3) top and bottom walls heated, side walls adiabatic. Corresponding experiments are carried out using water as the test fluid. The experimental results for water show that when the upper wall is heated the influence of buoyancy force is minimal. However, in the case of oil the temperature-sensitive viscosity results in enhanced heat transfer as compared to water due to the distortion of the oil velocity profile. When the lower wall is heated, the influence of both buoyancy force and the stress differences of the mineral oil caused by the variable viscosity results in a substantial increase in the heat transfer as compared to the values found for water. When both top and bottom walls are heated, the local heat transfer enhancement for the mineral oil is smaller as compared with that when each wall is heated alone since the velocity profile and the stress distribution associated with the variable viscosity are more symmetric. Thus the secondary flows are limited to those associated with natural convection and the oil results are in good agreement with the values found in water.

1. INTRODUCTION

IT IS KNOWN that for most high Prandtl number fluids, the physical properties of specific heat, thermal conductivity and density are nearly independent of temperature, but the viscosity decreases markedly with increasing temperature. In this case, when the difference between the fluid bulk temperature and the wall temperature is high, the property variation will alter the velocity and temperature profiles. Consequently the heat transfer and friction coefficients will be different from the values obtained if the properties were constant. An empirical method widely used to account for the temperature effect is the viscosity ratio method which can be described by following equations:

$$Nu/Nu_{cp} = (\eta_b/\eta_w)^n \quad (1)$$

and

$$f/f_{cp} = (\eta_b/\eta_w)^m. \quad (2)$$

The influence of variable properties on the flow and heat transfer differs in magnitude for different flow channel geometries. For circular duct flow, Deissler [1] and Shannon and Depew [2] numerically studied the influence of variable viscosity on fully-developed heat transfer with constant heat flux. They reported that the n value in equation (1) was 0.14. Yang [3] studied both the developing and fully developed region with both constant temperature and constant heat flux on the wall. The n value that he obtained is equal to 0.11. Sieder and Tate [4] experimentally investigated the same problem and reported an n value of 0.14. Oskay and Kakac [5] investigated the heat transfer of mineral oil flowing through a circular pipe with constant heat flux on the wall. Their result showed that the exponent m of the viscosity ratio should be -0.152. All the results mentioned above showed that the maximum influence of variable viscosity on Nusslet numbers and friction factors is generally less than 20% for circular channel flows, when the temperature difference between a test fluid and heated wall(s) is up to 30°C.

For flow in a rectangular duct with one wall heated only, the variable viscosity can cause unequal stresses on different sides of the wall. Butler and McKee [6] worked out an exact solution of the velocity distribution for fully developed flow of temperature dependent viscous fluids in heated rectangular ducts. Constant heat flux was imposed on the top wall of the duct with aspect ratios of 0.5, 5 and 10. They found that the temperature dependent viscosity resulted in a significant velocity profile distortion, i.e. the maximum velocity shifted well off center toward the hotter wall. They pointed out that such distortion might trigger instability at a lower Reynolds number. The average wall stress will have a different value for the heated and unheated walls which would tend to produce internal rotation in the flow.

In order to study the influence of variable viscosity on heat transfer in a rectangular duct, some heat trans-

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NOMENCLATURE

- C_p specific heat of fluid [J kg^{-1} K^{-1}]
- D_h hydraulic diameter of the duct [m]
- f Fanning friction factor, $\tau_w/(\rho u_m^2/2)$
- Gr Grashof number, $\rho^2 g\beta\Delta T D_h^3/\eta^2$
- Gr^* Grashof number, $\rho^2 g\beta q'' D_h^4/k\eta^2$
- Gz Graetz number, $\dot{m}C_p/kx$
- h local convective heat transfer coefficient, $q''/(T_w - T_b)$ [W m^{-2} K^{-1}]
- H1(1L) thermal boundary conditions representing constant wall heat flux axially and constant temperature peripherally on one longer wall of the rectangular duct
- H1(2L) thermal boundary conditions representing constant wall heat flux axially and constant temperature peripherally on two longer walls of the rectangular duct
- H1(4) thermal boundary conditions representing constant wall heat flux axially and constant temperature peripherally on all four walls of the rectangular duct
- k thermal conductivity [W m^{-1} K^{-1}]
- $\dot{m}$ mass flow rate [kg s^{-1}]
- Nu local Nusselt number, hD_h/k
- Nu_{L1} local Nusselt number for lower wall in the case of H1(1L) boundary condition
- Nu_{u1} local Nusselt number for upper wall in the case of H1(1L) boundary condition
- Nu_m mean value of upper wall Nusselt number and lower wall Nusselt number in the case of both walls heated with H1(2L) thermal boundary condition
- Pr Prandtl number, $\eta C_p/k$
- q'' heat flux per unit heating area [W m^{-2}]
- Ra_q Rayleigh number, $Gr^* Pr$
- Re Reynolds number, $\rho u_m D_h/\eta$
- Re^* Kozicki generalized Reynolds number, $1.029Re$
- T temperature [°C]
- T_b local fluid bulk temperature [°C]
- T_c characteristic temperature [°C]
- T_w local wall temperature of the duct [°C]
- u velocity components in axial direction [m s^{-1}]
- u_m mean velocity in axial direction [m s^{-1}]
- x axial rectilinear coordinate, or axial location from the duct entrance [m].

Greek symbols

- β volumetric coefficient of thermal expansion [K^{-1}]
- η Newtonian fluid viscosity [N s m^{-2}]
- ρ density of fluid [kg m^{-3}]
- τ_w shear stress at wall [N m^{-2}].

Subscripts

- b evaluated at mean bulk temperature
- cp corresponding to constant property conditions
- w evaluated at wall temperature.

fer experiments with mineral oil in a 2:1 rectangular duct with H1 thermal boundary conditions were carried out.

2. EXPERIMENTAL SYSTEM

The main functions of the experimental set-up are to store the test fluids, pump the fluids through a heat transfer test section, recover the heat gain of the test fluids by a heat exchanger and, finally, either let the fluids go down a drain or return them to the storage tanks for recirculation.

Figure 1 provides a schematic of the flow system, which consists of a reservoir, two auxiliary plastic tanks, a moyno positive displacement pump, a test section, a heat exchanger and a weighing tank. The test section has a rectangular cross section with an aspect ratio of 0.5 (1.8 cm × 0.9 cm) as seen in Fig. 2. The hydraulic diameter of the test section is 1.2 cm. The length of the duct is 640 cm, representing 533 hydraulic diameters which should be long enough to get fully developed flow. The calming section located before the test section is used to provide a flat velocity profile at the entrance of the test section. The function of the mixing section located after the test section is to mix the exiting fluid and provide an equalized bulk fluid temperature. Individual thermocouples are mounted at the calming section and the mixing section to measure the inlet and outlet bulk fluid temperatures.

The wall temperatures were measured by eighty-four calibrated 30 gauge T-type thermocouples located at 23 axial positions along the duct. Pressure taps of 0.078 in. (0.198 cm) i.d. were located at 16 positions along one plastic side wall to measure pressure drops. In this program, only local pressure drop under fully developed conditions was measured by a manometer.

The power supply provides a heating system with a maximum DC power output of 24 kW (2000 amp × 12 volt). The entire rectangular duct was insulated by a 12 in. × 12 in. (30.5 cm × 30.5 cm) plywood box filled

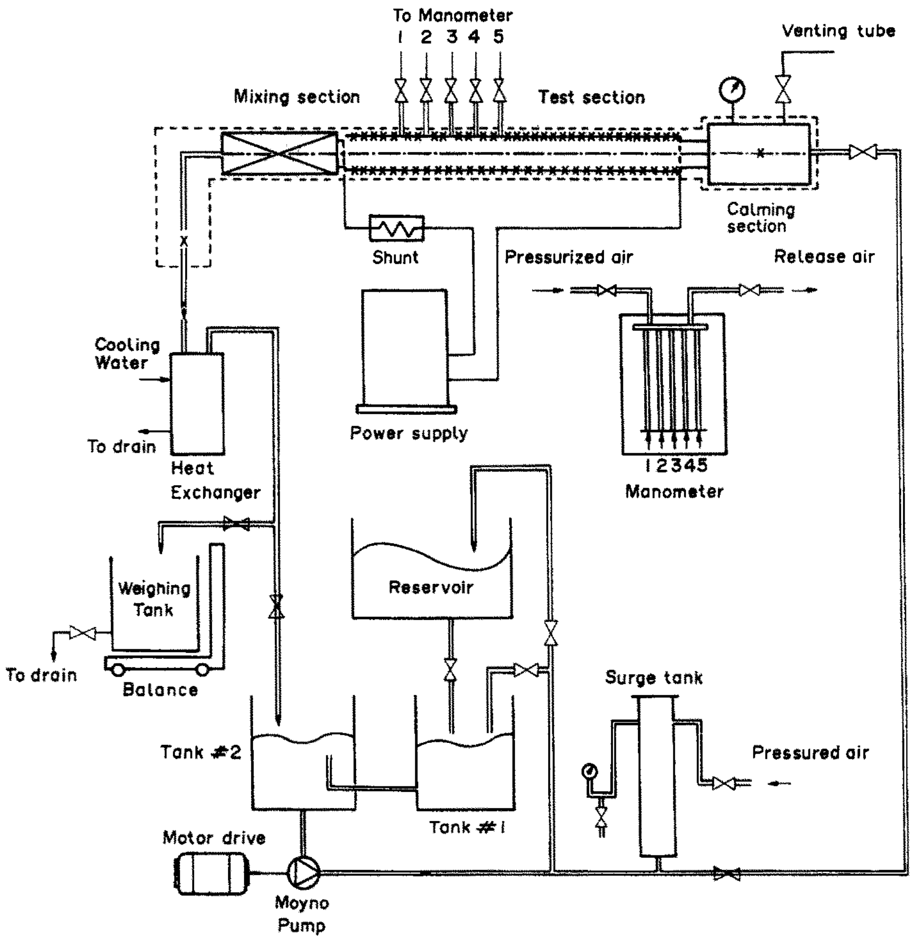

FIG. 1. Experimental set up.

with styrofoam beads having an effective thermal conductivity of 0.036 W m^{-1} K^{-1}.

3. PHYSICAL PROPERTIES OF MINERAL OIL

The properties of density, thermal conductivity, specific heat and viscosity of mineral oil were measured in the laboratory. They are shown in Fig. 3.

It can be seen from the figure that the density, thermal conductivity and specific heat of mineral oil do not vary appreciably with changes in temperature. However the viscosity of mineral oil is very sensitive to temperature. As a comparison, Table 1 lists the property ratios for water and mineral oil at temperatures of 20°C and 55°C which cover the range of heat transfer experiments.

The data in the table show that all the property changes over the specified temperature range are within 6% except for the viscosities. The viscosity ratio for water is 1.91 and up to 4.01 for mineral oil.

4. EXPERIMENTAL RESULTS

4.1. *Laminar friction factors of mineral oil*

As pointed out in the last section, the viscosity of the oil changes with temperature dramatically. Using the bulk fluid temperature as the characteristic temperature to determine the properties of the fluid results in a considerable departure of the experimental data from the constant property prediction by Kozicki *et al.* [7], $f = 16/Re^*$. Better correspondence of the experimental friction factors with the constant properties prediction can be achieved if a mean value between the wall temperature and the bulk fluid temperature is used as the characteristic temperature of the oil. This temperature can be described as

$$T_c = (T_b + T_w)/2 \quad (3)$$

where T_b is the bulk fluid temperature, T_w is the wall temperature and T_c is the characteristic temperature. The measured friction factor results, based on the above defined T_c for all three thermal boundary con-

Table 1. Property ratios of water and mineral oil at temperatures of 20°C and 55°C

Fluid	Properties ρ_{20}/ρ_{55}	k_{20}/k_{55}	C_{p20}/C_{p55}	η_{20}/η_{55}
Water	1.01	0.93	1.00	1.91
Mineral oil	1.03	0.96	0.94	4.01

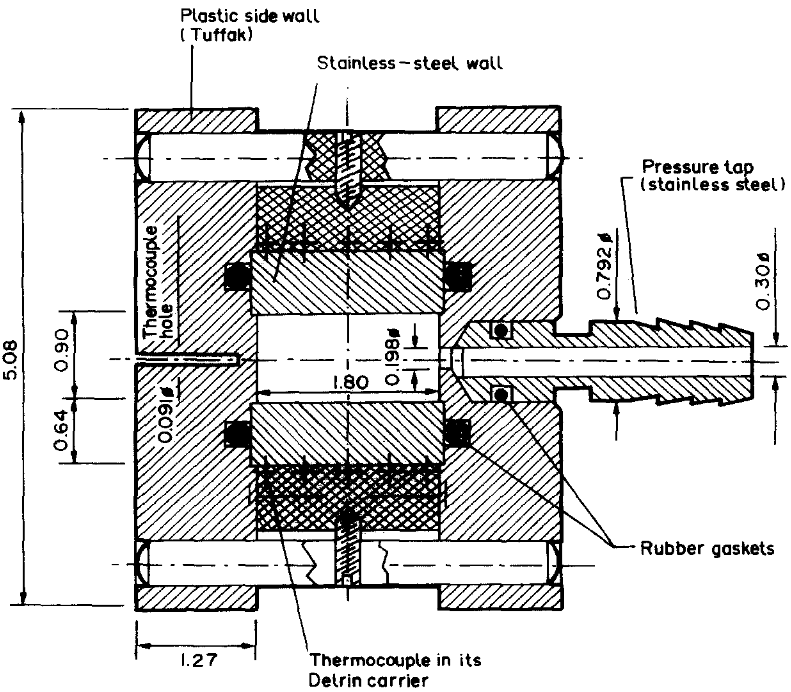

FIG. 2. Test section.

ditions, are shown in Fig. 4. Good agreement between the experimental data and the analytical solution was obtained. The relative deviations are less than 10% for all runs.

4.2. *Forced convection of mineral oil*

The initial heat transfer experiments with oil attempted to minimize free convection and consequently the upper wall is the only heated wall in these runs. The Reynolds numbers range from 32 to 1300. Typical results are shown in Fig. 5. Also shown in the figure are the span of heat transfer measurements reported for water by Xie [8] for the same boundary conditions. The prediction of Wibulswas [9] for the case where all four walls are heated, with simultaneous development of the thermal and hydrodynamic boundary conditions, for a fluid Prandtl number of 10 is also shown. As expected the one-wall heated experimental Nusselt values for water fall below the prediction of Wibulswas since the limiting Nusselt number for the H1(4) condition is equal to 4.12 while 3.53 is the limiting Nusselt number for the H1(1L) boundary condition.

Compared with the experimental results found for water the Nusselt numbers of the mineral oil are about 30–50% higher. It is hypothesized that this increase results from the variable viscosity due to the large temperature difference of the fluid over the cross-section of the duct. This viscosity variation causes the maximum velocity to shift off the center toward the hotter wall. As a result, the average wall stress will have different values for the hot, cold and side walls. These unequal stresses produce an internal rotation (i.e. secondary motion) in the flow. When the Reynolds numbers are small, this secondary flow enhances the heat transfer. This also occurs when the Rayleigh numbers are very large, in which case the temperature variations are large and the variable viscosity induced secondary flow intensifies. Both the Reynolds number effect and the Rayleigh number effect on the heat transfer are shown in Fig. 5.

It can be seen from the figure that increasing the Rayleigh number and decreasing the Reynolds number causes the heat transfer to increase. However, at relatively higher Reynolds numbers and lower Rayleigh numbers (as in the case of run #4) the oil heat transfer results are very close to those found for water. This suggests that the Prandtl number does not influence the heat transfer behavior except that it increases the length of the thermal developing region. Compared with the water results at the same Reynolds number, the Graetz number of mineral oil (proportional to the Prandtl number) is increased by a factor of twenty. Therefore a longer x/D_h is needed for a higher Prandtl number fluid, such as oil, to reach a fully developed flow condition.

Laminar flow in a 2:1 rectangular duct with the upper wall heated was also studied numerically by Butler and McKee for fluid with temperature-sensitive viscosity [6]. They found that the velocity profile was distorted due to the variable viscosity. Their analysis and conclusions are consistent with the present results.

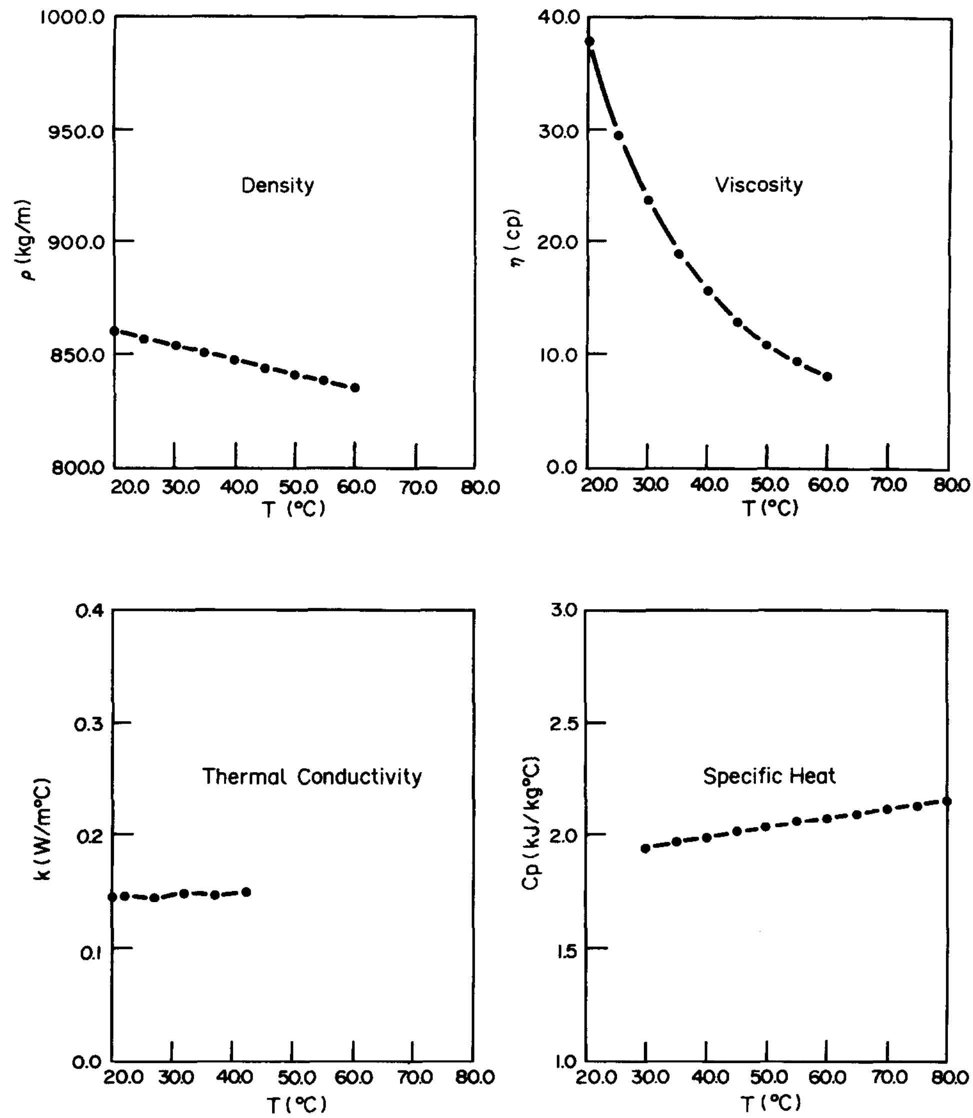

FIG. 3. Physical properties of mineral oil.

4.3. *Mixed convection of mineral oil*

Mixed convection was realized by heating the lower wall alone or by heating simultaneously the upper and lower walls of the duct. In contrast with the case of purely forced convection, the heat transfer behavior in these cases was influenced by both the variable viscosity and the buoyancy force.

Under mixed convection conditions the parameter Gr^*/Re^2 is used to represent the ratio of buoyancy force and inertia force. If the magnitude of this parameter is much greater than one, free convection dominates the heat transfer. If the number is much less than one, inertia force dominates. If the number is equal to one, then both free and forced convection play roles.

Figure 6 shows that when Gr^*/Re^2 is close to 1, the free convection effect on the heat transfer appears (i.e. run #2 and run #20). However, compared with water for a given Ra_q and Gr^*/Re^2, the Nusselt numbers of oil are higher than those of water. This gives further proof that the secondary flow caused by the variable viscosity is superimposed on the free convection in this case.

When both top and bottom walls are heated, the variable viscosity is not as important as when only one wall is heated. Since both upper and lower walls are heated symmetrically, the velocity distribution tends to be more symmetrical than in the case where only one wall is heated. As a result the secondary flow is avoided and the heat transfer enhancement is not as high as in the case where one wall is heated alone. Comparing Fig. 7 with Fig. 6, when two walls are heated, the oil heat transfer coefficients on the lower wall are not as high as when the lower wall was

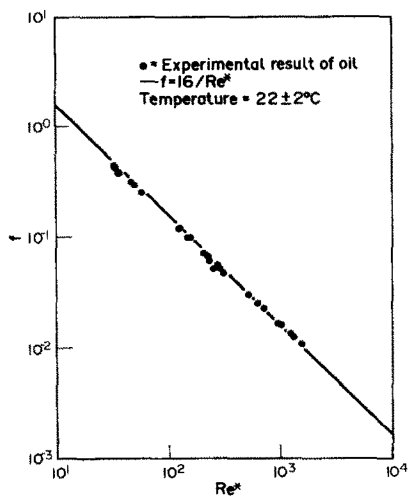

FIG. 4. Friction factor of mineral oil in a 2:1 duct.

heated alone. The experimental heat transfer values for the mineral oil on the upper wall are very close to the predicted forced convection limit under the H1(2L) boundary condition. This indicates that forced convection plays the dominate role on the top wall.

5. CONCLUSIONS

Experimental heat transfer results are presented for the laminar flow of a mineral oil with temperature-sensitive viscosity in a 2:1 rectangular duct. The H1 thermal boundary condition corresponding to axially constant heat flux and peripherally constant temperature was studied for these different heating configurations: (1) top wall heated, other walls adiabatic; (2) lower wall heated, other walls adiabatic; (3) top and bottom walls heated, side walls adiabatic. The results are compared with the heat transfer values found for water under the same thermal boundary conditions.

For the case where the upper wall only is heated the local Nusselt values for oil are 30 to 40% higher than the values for water. This is ascribed to the influence of a secondary flow resulting from an asymmetric velocity associated with the variable viscosity of the mineral oil.

For the case where the lower wall only is heated the influence of variable viscosity in the case of the mineral oil results in a similar secondary flow which is superimposed on the free convection. As a result, the local heat transfer values for oil are substantially higher than those found for water.

For symmetrical heating the heat transfer results for the mineral oil are in agreement with the results found for water. This is explained by the fact that the symmetric heating generates a symmetric velocity profile and the secondary flows associated with the variable viscosity are reduced.

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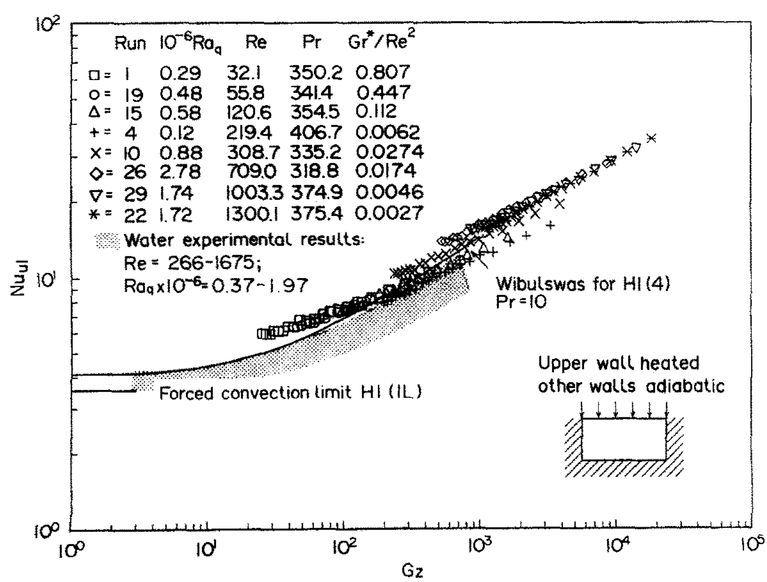

FIG. 5. Laminar heat transfer of mineral oil in a 2:1 duct with upper wall heated.

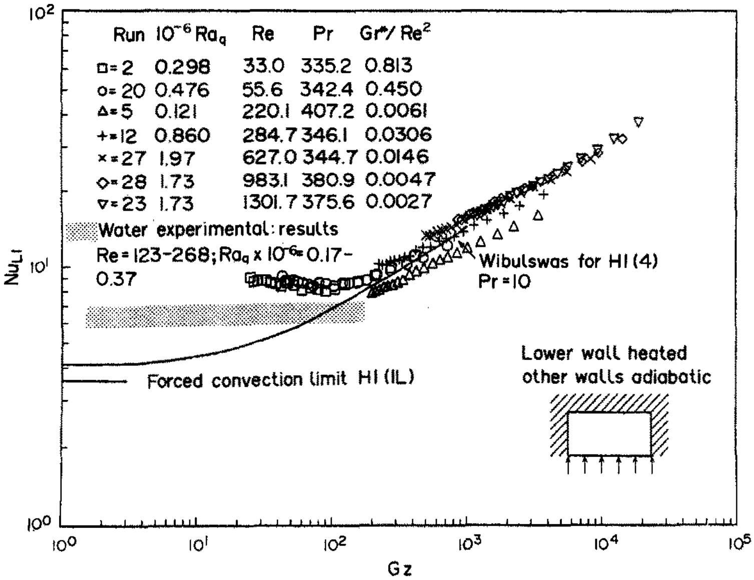

FIG. 6. Laminar heat transfer of mineral oil in a 2:1 duct with lower wall heated.

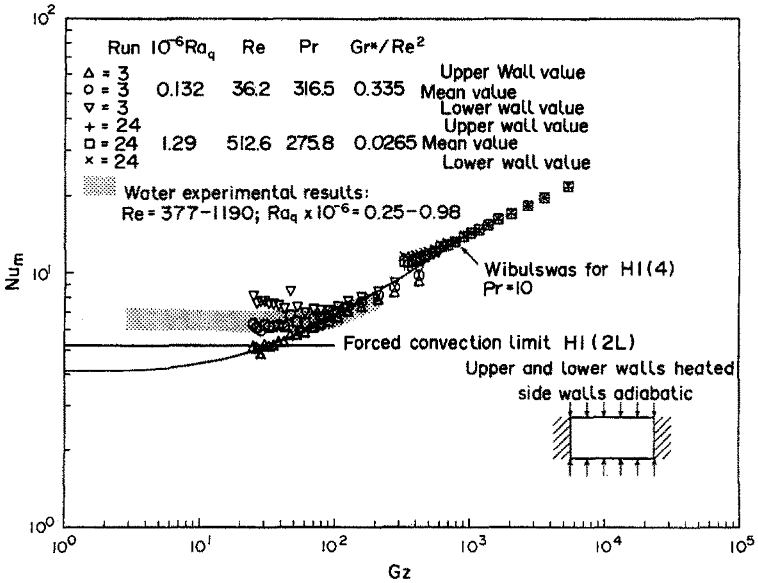

FIG. 7. Laminar heat transfer of mineral oil in a 2:1 duct with both upper and lower wall heated.

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INFLUENCE DE LA VISCOSITE VARIABLE D'UNE HUILE MINERALE SUR LE TRANSFERT THERMIQUE LAMINAIRE DANS UN CANAL RECTANGULAIRE 2:1

Résumé—On conduit une étude expérimentale du transfert thermique laminaire dans un canal rectangulaire 2:1 avec de l'huile minérale. La condition H1 de limite thermique correspond à un flux thermique constant axialement et à une température uniforme sur la périphérie; elle est adoptée pour trois configurations différentes de chauffage: (1) paroi supérieure chauffée, les autres parois étant adiabatiques, (2) paroi inférieure chauffée avec les autres parois adiabatiques, (3) parois supérieure et inférieure chauffées avec les parois latérales adiabatiques. Les expériences utilisent l'eau comme fluide d'essai. Les résultats expérimentaux avec l'eau montrent que lorsque la paroi supérieure est chauffée l'influence du flottement est minimale. Néanmoins dans le cas de l'huile, la variation de la viscosité avec la température augmente le transfert en comparaison avec le cas de l'eau à cause de la distorsion du profil de vitesse de l'huile. Quand la paroi inférieure est chauffée, l'influence combinée de la variation de viscosité et sur la force de flottement et sur les différences de contraintes conduit à un accroissement substantiel du transfert de chaleur par rapport au cas de l'eau. Lorsque les parois supérieure et inférieure sont chauffées, l'augmentation du transfert thermique local pour l'huile est plus faible que si une seule des parois était chauffée parce que le profil des vitesses et la distribution des contraintes sont plus symétriques. Ainsi les écoulements secondaires sont limités à ceux associés à la convection naturelle et les résultats avec l'huile sont en bon accord avec les valeurs trouvées pour l'eau.

EINFLUSS DER VARIABLEN VISKOSITÄT EINES MINERALÖLS AUF DEN WÄRMEÜBERGANG BEI LAMINARER STRÖMUNG IN EINEM RECHTECKKANAL MIT DEM SEITENVERHÄLTNIS 2:1

Zusammenfassung—Der Wärmeübergang bei laminarer Strömung von Mineralöl in einem Rechteckkanal mit dem Seitenverhältnis 2:1 wird experimentell untersucht. Bei konstanter Wärmestromdichte in Längsrichtung und konstanter Temperatur in Umfangsrichtung werden drei unterschiedliche Fälle betrachtet: (1) Die obere Wand wird beheizt, die übrigen Wände sind adiabat; (2) die untere Wand wird beheizt, die übrigen Wände sind adiabat; (3) die obere und untere Wand wird beheizt, die Seitenwände sind adiabat. Die entsprechenden Experimente wurden zusätzlich auch mit Wasser als Versuchsstoff durchgeführt. Die Ergebnisse für Wasser zeigen, daß für eine Beheizung von oben Auftriebseffekte minimal sind. Im Fall von Öl führt jedoch die temperaturempfindliche Viskosität zu einer Verbesserung des Wärmeübergangs im Vergleich zu Wasser, was auf eine Störung des Geschwindigkeitsprofils bei Öl zurückgeführt wird. Bei einer Beheizung von unten ist der Wärmeübergang in Öl spürbar besser als in Wasser. Dies wird auf die Auftriebskräfte und auf unterschiedliche Schubspannungen im Mineralöl aufgrund der variablen Viskosität zurückgeführt. Wenn von oben und von unten geheizt wird, ist die Erhöhung des örtlichen Wärmeübergangs bei Mineralöl kleiner als bei der einseitigen Beheizung. Die Ursache dafür wird darin gesehen, daß das Geschwindigkeitsprofil und die Verteilung der Schubspannung aufgrund der variablen Viskosität stärker symmetrisch sind. Daher kommt es nur zu Sekundärströmungen infolge natürlicher Konvektion, und die Ergebnisse mit Öl stimmen gut mit denjenigen für Wasser überein.

ВЛИЯНИЕ ПЕРЕМЕННОЙ ВЯЗКОСТИ НЕФТИ НА ТЕПЛОПЕРЕНОС ПРИ ЛАМИНАРНОМ ТЕЧЕНИИ В КАНАЛЕ ПРЯМОУГОЛЬНОГО СЕЧЕНИЯ С ОТНОШЕНИЕМ СТОРОН 2:1

Аннотация—Экспериментально исследуется теплоперенос нефти при ламинарном течении в прямоугольном канале с отношением сторон, равным 2 : 1. Для трех различных конфигураций нагрева, а именно, 1) с нагретой верхней стенкой и остальными адиабатическими, 2) с нагретой нижней стенкой и остальными адиабатическими, 3) с нагретыми верхней и нижней стенкой и адиабатическими боковыми было принято тепловое граничное условие, соответствующее постоянному аксиальному потоку и постоянной по периметру температуре. Эксперименты проводились с использованием воды в качестве рабочей жидкости. Полученные для воды экспериментальные результаты показали, что при нагреве верхней стенки влияние подъемной силы минимально. Однако для нефти зависящая от температуры вязкость вызывала увеличение теплопереноса по сравнению с водой в силу искажения брофиля скоростей нефти. При нагреве нижней стенки влияние подъемной силы и напряжений в случае нефти, вызванное изменением вязкости, обусловливало существенное увеличение теплопереноса по сравнению со значениями, полученными для воды. При нагреве как верхней, так и нижней стенок локальный теплоперенос в случае с нефтью интенсифицировался меньше, чем при нагреве одной из стенок, т.к. связанные с изменяющейся вязкостью профиль скоростей и распределение напряжений были более симметричны. Таким образом, вторичные течения связаны с естественной конвекцией.